ENERGY DELAY TRADE-OFF

Trading delay for energy is another source of energy reduction. In general, for the fixed-input fixed-output condition, the best performance of the circuit occurs when its energy sensitivity is the greatest. A large amount of energy can potentially be saved if the delay is relaxed. Shown below is the method that yields the most energy reduction for the given slack of the delay.

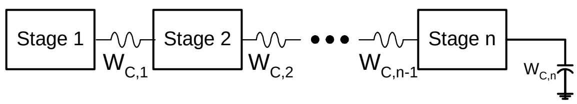

Figure 1: n-stage datapath including wire capacitance

Assume a datapath consisting of m-input n-stage multi-path circuit, Figure 1. For simplicity, identical set of gates (such as multiplexers) are assumed for the 1st-stage; these gates are called drivers.

The circuit must drive a fixed output load C_{Load} and is represented by an equivalent gate size $W_{C,n}$. In addition, possible wire capacitance between stages i and (i+1) is expressed by $W_{C,i}$.

The solution for the best delay may be obtained using equal stage effort *f* for all stages.

Straightforwardly, the size of gate k in final stage n is given by

$$W_{n,k} = \frac{g_{n,k} W_{C,n}}{f} \tag{1}$$

Similarly, the size of an arbitrary gate k in stage (n-1) is computed with the load from gates in stage n and the load by the global wire capacitance $W_{C,n-1}$.

$$\begin{aligned} W_{n-1,k} &= \frac{g_{n-1,k}\left(\sum_p \chi_p W_{n,p} + W_{C,n-1}\right)}{f} \\ &= g_{n-1,k}\left(\sum_p \chi_p g_{n,p}\right)\frac{W_{C,n}}{f^2} + g_{n-1,k}\frac{W_{C,n-1}}{f} \\ &= \zeta_{n-1,0,k}\frac{W_{C,n}}{f^2} + \zeta_{n-1,1,k}\frac{W_{C,n-1}}{f} \end{aligned} \tag{1.1}$$

where

$$\zeta_{n-1,0,k} = g_{n-1,k}\left(\sum_p \chi_p g_{n,p}\right)$$

$$\zeta_{n-1,1,k} = g_{n-1,k}$$

are the portions of gate size set by the output load and the global wire capacitance, respectively.

In general, the size of any gate k in stage i can be expressed as

$$W_{i,k} = \sum_{j=i}^{n} \zeta_{i,n-j,k} \frac{W_{C,j}}{f^{n+1-j}} \tag{1.i}$$

The $\zeta_{i,n-j,k}$ terms are constant factors and computable for the given design.

In addition, the load to the 1st-stage driver is

$$W_{driver,Load} = \sum_k W_{2,k} = W_{driver,Load} \sum_k \sum_{j=2}^{n} \rho_{n-j,k}$$
$$\text{with } \rho_{n-j,k} = \zeta_{2,n-j,k} \frac{W_{C,j}}{f^{n+1-j}}$$

The delay effort of the driver stage is

$$\frac{W_{driver,Load}}{W_{in}} = f$$

The total delay of the datapath is

$$T_{d,Total} = \sum_{i=1}^{n} (f + p_i) \tag{2}$$

Using equation (d) in the Energy Estimation section,, the total energy is given by

$$E_{Total} = \sum_{i=1}^{n}\sum_{k} c_{g(i,k)} W_{g(i,k)} + E_{wire} \tag{3}$$

$$= E_1 + \sum_{i=2}^{n}\sum_{k}\left(c_{g(i,k)} \sum_{j=i}^{n} \zeta_{i,n-j,k} \frac{W_{C,j}}{f^{\,n+1-j}} \right) + E_{wire}$$

$$= E_1 + \sum_{i=2}^{n}\sum_{j=i}^{n}\left(\sum_{k} c_{g(i,k)} \zeta_{i,n-j,k} \frac{W_{C,j}}{f^{\,n+1-j}} \right) + E_{wire}$$

$$= E_1 + \sum_{i=2}^{n}\sum_{j=i}^{n} E_{i,n-j} + E_{wire}$$

where

$$E_{i,n-j} = \left(\sum_{k} c_{g(i,k)} \zeta_{i,n-j,k} \right) \frac{W_{C,j}}{f^{\,n+1-j}}$$

The energy term $E_{i,n-j}$ refers to the energy consumption for the gate in stage i that is influenced by the output load or global wire capacitance $W_{C,j}$.

To achieve more efficient energy, a delay of ΔT_d is traded and is distributed to the stage effort of each stage via the factor y_i ($i = 2 \rightarrow n$) such that $f_i = y_i f$. Note that the delay in stage 1 is now determined by the new load from stage 2.

It can be shown that the new gate size is

$$W_{i,k}\{ y \} = \sum_{j=i}^{n} \zeta_{i,n-j,k} \frac{W_{C,j}}{f^{\,n+1-j} \prod_{p=i}^{j} y_p} \tag{4}$$

The new load to the driver is

$$W_{driver,Load}\{ y \} = W_{driver,Load} \sum_{k}\sum_{j=2}^{n} \frac{\rho_{j,k}}{\prod_{p=j}^{n} y_p}$$

and the new stage effort in the driver is

$$\frac{W_{driver,Load}\{ y \}}{W_{in}} = f \sum_{k}\sum_{j=2}^{n} \frac{\rho_{j,k}}{\prod_{p=j}^{n} y_p}$$

The new total delay of the datapath is

$$T_{d,Total}\{ y \} = \frac{W_{driver,Load}}{W_{in}} + p_1 + \sum_{i=2}^{n} \left(y_i f + p_i \right) \tag{5}$$

$$= (f \sum_{k}\sum_{j=2}^{n} \frac{\rho_{n-j,k}}{\prod_{p=j}^{n} y_p} + p_1) + \sum_{i=2}^{n} \left(y_i f + p_i \right)$$

The equation for the traded delay is

$$\Delta T_d = T_{d,Total}\{ y \} - T_{d,Total}$$

$$= f(\sum_{k}\sum_{j=2}^{n} \frac{\rho_{n-j,k}}{\prod_{p=j}^{n} y_p} - 1 + \sum_{i=2}^{n} y_i)$$

or

$$\sum_{k}\sum_{j=2}^{n} \frac{\rho_{j,k}}{\prod_{p=j}^{n} y_p} - 1 + \sum_{i=2}^{n} y_i - \frac{\Delta T_d}{f} = 0 \equiv C \tag{6}$$

Equation (6) is called the energy delay tradeoff C.

Note that $y_i > 0$ so that $f_i > 0$ ($i=1 \rightarrow n$). The upper bound of each y_i is determined by (6).

The corresponding total energy is

$$E_{Total}\{ y \} = E_1 + \sum_{i=2}^{n}\sum_{j=i}^{n} \frac{E_{i,n-j}}{\prod_{p=i}^{j} y_p} + E_{wire} \tag{7}$$

The optimal energy can be found by solving the LaGrange function

$$F = \lambda C + E_{Total} \tag{8}$$

$$= \lambda(\sum_{k}\sum_{j=2}^{n} \frac{\rho_{n-j,k}}{\prod_{p=j}^{n} y_p} - 1 + \sum_{i=2}^{n} y_i - \frac{\Delta T_d}{f}) +$$

$$E_1 + \sum_{i=2}^{n}\sum_{j=i}^{n} \frac{E_{i,n-j}}{\prod_{p=i}^{j} y_p} + E_{wire}$$

The solution is obtained from (6) and n-1 derivatives, $\partial F / \partial y_i = 0$ for $i=2 \rightarrow n$. The n-1 derivatives can be simplified into

$$\lambda\left(-\sum_k\sum_{j=2}^{2}\frac{\rho_{n-j,k}}{y_2\prod_{p=j}^{n}y_p}+1\right)-\sum_{i=2}^{2}\sum_{j=2}^{n}\frac{E_{i,n-j}}{y_2\prod_{p=i}^{j}y_p}=0 \tag{9.1}$$

$$(y_3-y_2)\left(\lambda\sum_k\sum_{j=2}^{2}\frac{\rho_{n-j,k}}{\prod_{p=j}^{n}y_p}+\sum_{i=2}^{2}\sum_{j=3}^{n}\frac{E_{i,n-j}}{\prod_{p=i}^{j}y_p}\right)+y_3\sum_{i=2}^{2}\sum_{j=2}^{2}\frac{E_{i,n-j}}{\prod_{p=i}^{j}y_p} \tag{9.2}$$

$$=y_2\left(\lambda\sum_k\sum_{j=3}^{3}\frac{\rho_{n-j,k}}{\prod_{p=j}^{n}y_p}+\sum_{i=3}^{3}\sum_{j=3}^{n}\frac{E_{i,n-j}}{\prod_{p=i}^{j}y_p}\right)$$

$$(y_4-y_3)\left(\lambda\sum_k\sum_{j=2}^{3}\frac{\rho_{n-j,k}}{\prod_{p=j}^{n}y_p}+\sum_{i=2}^{3}\sum_{j=4}^{n}\frac{E_{i,n-j}}{\prod_{p=i}^{j}y_p}\right)+y_4\sum_{i=2}^{3}\sum_{j=3}^{3}\frac{E_{i,n-j}}{\prod_{p=i}^{j}y_p} \tag{9.3}$$

$$=y_3\left(\lambda\sum_k\sum_{j=4}^{4}\frac{\rho_{n-j,k}}{\prod_{p=j}^{n}y_p}+\sum_{i=4}^{4}\sum_{j=4}^{n}\frac{E_{i,n-j}}{\prod_{p=i}^{j}y_p}\right)$$

(…)

$$(y_n-y_{n-1})\left(\lambda\sum_k\sum_{j=2}^{n-1}\frac{\rho_{n-j,k}}{\prod_{p=j}^{n}y_p}+\sum_{i=2}^{n-1}\sum_{j=n}^{n}\frac{E_{i,n-j}}{\prod_{p=i}^{j}y_p}\right)+y_n\sum_{i=2}^{n-1}\sum_{j=n-1}^{n-1}\frac{E_{i,n-j}}{\prod_{p=i}^{j}y_p} \tag{9.n-1}$$

$$=y_{n-1}\left(\lambda\sum_k\sum_{j=n}^{n}\frac{\rho_{n-j,k}}{\prod_{p=j}^{n}y_p}+\sum_{i=n}^{n}\sum_{j=n}^{n}\frac{E_{i,n-j}}{\prod_{p=i}^{j}y_p}\right)$$

Equations (6) and (9.1)–(9.n-1) are solvable, using the numerical method. The run time remains O(n) where n is the number of stages.